



Caboto

PRICING & HEDGING INTEREST RATE DERIVATIVES

Marco Bianchetti

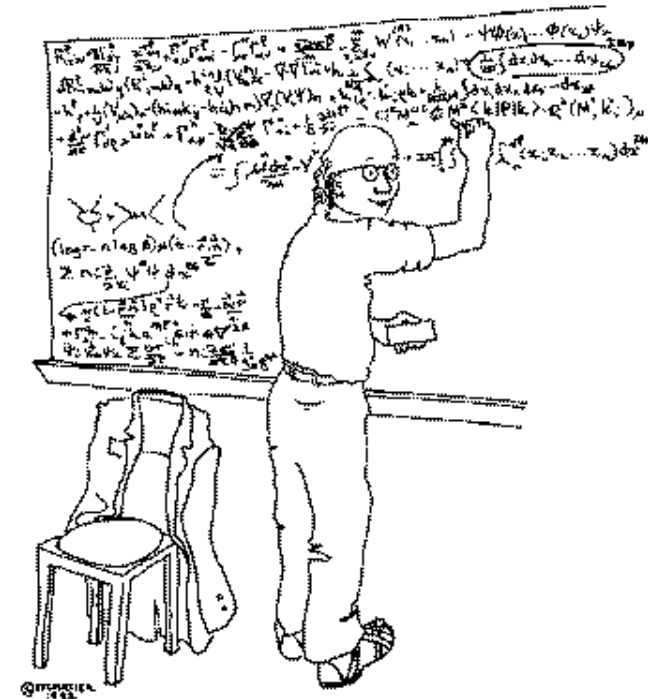
Financial Engineering – Derivatives Modelling

marco.bianchetti@caboto.it

Summary

- **Definition of the business problem**
 - Example: “*Schermo Totale*”
- **One model**
 - Basic assumptions
 - Solution in closed form
 - Calibration
- **Results**
 - Forward curve & pricing
 - Greek letters & bookkeeping
- **From theory to trading**
 - The process & the characters
 - Organisation
 - Computing
 - Platforms, technologies, architectures
- **References**

Some math will be necessary...



“At this point we notice that this equation is beautifully simplified if we assume that space-time has 92 dimensions.”

Definition of the business problem

A) Example: "Schermo Totale" - termsheet



Deal characteristics:

- start: 30 Jan. 2003
- maturity: 30 Jan. 2006 (3Y)
- nominal: 2×10^8 €
- underlying CPI: *HICP unrevised (Bloomberg: CPALEMU)*
- reference CPI fixing : Oct. 2003

Counterparty A pays to counterparty B:

- upfront: XXX % (*unknown to be determined*)
- I coupon: 3.5% (*% annual, act/act, unadj.*)
- II+III coupon: $0.80\% + \text{MAX}[0\%; 100\% \times (\text{HICP}_f / \text{HICP}_{i-1})]$ (*idem*)

Here is the income...

...and here is the risk!

Counterparty B pays to counterparty A:

- Euribor 3M + 24 bps (*% quarterly, act/360, adj.*)

"Structuration":
creare strumenti complessi
a partire da "mattoni" semplici

Payoff = Plain vanilla swap + inflation-linked swap + opzione

Definition of the business problem

B) "Schermo Totale" - inflation-linked cashflows



Inflation-linked cashflows

■ maturity cashflows:

□ 30 Jan. 2005: $\text{Max} \left[\frac{X_{\text{Oct.04}}}{X_{\text{Oct.03}}} - 1 ; 0 \right]$

□ 30 Jan. 2006: $\text{Max} \left[\frac{X_{\text{Oct.05}}}{X_{\text{Oct.04}}} - 1 ; 0 \right]$

■ Expected cashflows at settlement date (NPV):

□ 30 Jan. 2005:

$$d_{\text{settl}}(30/01/05) \times \left\{ \overset{\text{Expected value}}{E_{\text{settl}}^{30/01/05} \left[\frac{X_{\text{Oct.04}}}{X_{\text{Oct.03}}} - 1 \right]} + E_{\text{settl}}^{30/01/05} [\text{option}] \right\}$$

□ 30 Jan. 2006:

$$d_{\text{settl}}(30/01/06) \times \left\{ \overset{\text{Expected value}}{E_{\text{settl}}^{30/01/06} \left[\frac{X_{\text{Oct.05}}}{X_{\text{Oct.04}}} - 1 \right]} + E_{\text{settl}}^{30/01/06} [\text{option}] \right\}$$

Now we need a model!

One model

A) Basic assumptions

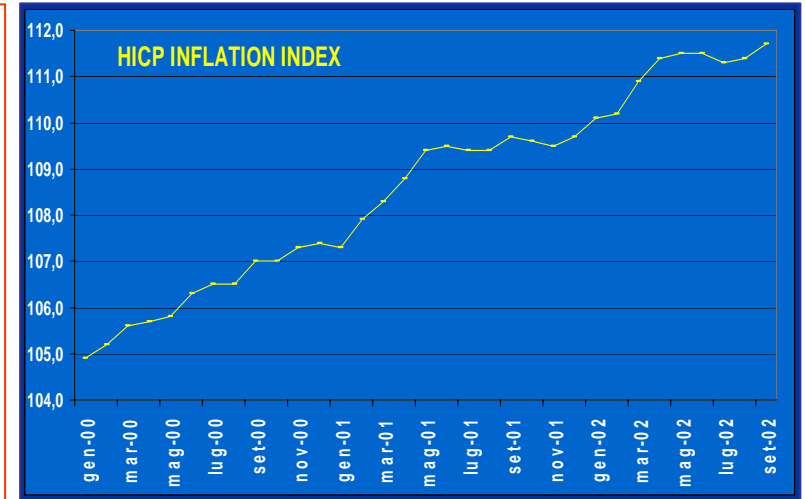


Assumptions on the underlying CPI X_t :

- **non-tradable**
- **monthly** fixing with **lag** δ
- **stochastic continuous** process
- **BS-type** diffusion dynamic:

$$\frac{dY_t}{Y_t} = \underbrace{\mu(t)}_{\text{drift}} dt + \underbrace{\sigma(t)}_{\text{volatility}} dW_t, \quad Y_t = X_{t-\delta}$$

brownian motion



Assumptions on the interest rate r_t :

- \exists a **bank account** $B(t)$, a **spot Martingale measure** and a **short rate** r_t s.t. $\forall T > t, \forall \delta_c > \delta$:
- **Hull-White** diffusion dynamic:

Exp. value at t

$$\frac{d_t(T)}{B(t)} = E_t \left[\frac{1}{B(T)} \right] = E_t \left[\exp \left(- \int_t^T r_u du \right) \right]$$

short rate

$$P_t(T, \delta_c) = B(t) E_t \left[\exp \left(- \int_t^T r_u du \right) X_{T-\delta_c} \right]$$

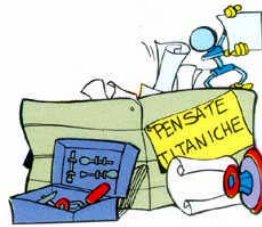
forward contract unknown

$$dr_t = \kappa \left[\vartheta(t) - r_t \right] dt + \xi dW_t$$

mean reversion volatility brownian motion

One model

B) Solution in closed form



N stochastic factors

- from spot to T-forward measure:

$$\begin{cases} \frac{dY_t}{Y_t} = \mu^T(t) dt + \sigma(t) \cdot dW_t^T, & dW_t^T = dW_t + \xi B(t, T) dt \\ \mu^T(t) = \mu(t) - \xi \sigma(t) B(t, T), \end{cases}$$

- Zero coupon price:**

$$P_t(T, \delta_c) = d_t(T) \left(\frac{1}{X_{t-\delta_c}} E_t^T [X_{T-\delta_c}] - 1 \right) = D_t(T) \left[\frac{X_{t-\delta}}{X_{t-\delta_c}} e^{\int_t^{T-(\delta_c-\delta)} \mu^T(u) du} - 1 \right]$$

End fixing, unknown (pointing to $X_{T-\delta_c}$)
Last published fixing, known (pointing to $X_{t-\delta}$)
Start fixing, known (pointing to $X_{t-\delta_c}$)

- Forward YoY coupon price:**

$$P_t(S, T, \delta_c) = d_t(T) E_t^T \left[\frac{X_{T-\delta_c}}{X_{S-\delta_c}} - 1 \right] = d_t(T) \left[\frac{P_t(T, \delta_c) D_t(S)}{P_t(S, \delta_c) D_t(T)} e^{-C_t(S, T, \delta_c, \sigma)} - 1 \right]$$

Start & end fixing, unknown (pointing to $X_{T-\delta_c}$)
Convexity adjustment, depends on volatility only (pointing to the exponential term)

- Call/put option price:**

$$P_t(T, \delta_c, K, \chi) = d_t(T) \text{Max} \left[\chi (X_{T-\delta_c}, K); 0 \right] = \dots \quad (\chi = \pm 1)$$

- Floored YoY coupon price:**

$$P_t(S, T, \delta_c, K) = d_t(T) \text{Max} \left[\frac{X_{T-\delta_c}}{X_{S-\delta_c}} - 1, K \right] = \dots$$



One model

C) Calibration [1]



Bootstrapping!

- 1st step: short rate calibration:
 - nominal mkt rates [$r_{mkt} \rightarrow \theta$]
 - nominal mkt cap/floor vols [$\sigma_{cap/floor} \rightarrow \kappa, \xi$]
- 2nd step: CPI volatility calibration:
 - mkt YoY inflation rates [$R_{YoY} \rightarrow \sigma(t)$]
- 3rd step: CPI drift calibration:
 - mkt ZC inflation rates [$R_{ZC} \rightarrow \mu(t)$]

Output:
 short rate
 mean reversion
 &
 volatility
 (H.W.)
 +
 CPI drift
 &
 volatility
 (B.S.)

HICP EU all items											
Mkt ZC Swap rates				Mkt YoY Swap Rates				Basis CPI			
Node	Bid (%)	Ask (%)	Mid (%)	Node	Bid (%)	Ask (%)	Mid (%)	Date	01/11/04		
1	5.2782	5.2782	5.2782	1	5.2782	5.2782	5.2782	Fixing	116.4		
2	-0.5059	-0.5059	-0.5059	2	-0.4888	-0.4888	-0.4888	Mkt calendar	EURO		
3	1.0349	1.0349	1.0349	3	1.0417	1.0417	1.0417	Mkt daycount	2		
4	2.3376	2.3376	2.3376	4	2.3202	2.3202	2.3202	Delay min.	59		
5	3.0997	3.0997	3.0997	5	3.0606	3.0606	3.0606				
6	2.7680	2.7680	2.7680	6	2.7387	2.7387	2.7387				
7	2.4927	2.4927	2.4927	7	2.4708	2.4708	2.4708				
8	1.9392	1.9392	1.9392	8	1.9321	1.9321	1.9321				
9	1.7219	1.7219	1.7219	9	1.7190	1.7190	1.7190				
10	1.7434	1.7434	1.7434	10	1.7389	1.7389	1.7389				
11	1.8759	1.8759	1.8759	11	1.8673	1.8673	1.8673				
12	1.8000	1.8000	1.8000	12	1.7929	1.7929	1.7929				
1	1.8000	1.8000	1.8000	1	1.8000	1.8000	1.8000				
2	2.1300	2.1300	2.1300	2	2.1300	2.1300	2.1300				
3	2.1600	2.1600	2.1600	3	2.1575	2.1575	2.1575				
4	2.1900	2.1900	2.1900	4	2.1875	2.1875	2.1875				
5	2.2300	2.2300	2.2300	5	2.2250	2.2250	2.2250				
6	2.2400	2.2400	2.2400	6	2.2350	2.2350	2.2350				
7	2.2700	2.2700	2.2700	7	2.2625	2.2625	2.2625				
8	2.2900	2.2900	2.2900	8	2.2825	2.2825	2.2825				
9	2.3100	2.3100	2.3100	9	2.3025	2.3025	2.3025				
10	2.3400	2.3400	2.3400	10	2.3300	2.3300	2.3300				
15	2.4200	2.4200	2.4200	15	2.4000	2.4000	2.4000				
20	2.5100	2.5100	2.5100	20	2.4650	2.4650	2.4650				
25	2.5700	2.5700	2.5700	25	2.5050	2.5050	2.5050				
30	2.6100	2.6100	2.6100	30	2.5375	2.5375	2.5375				

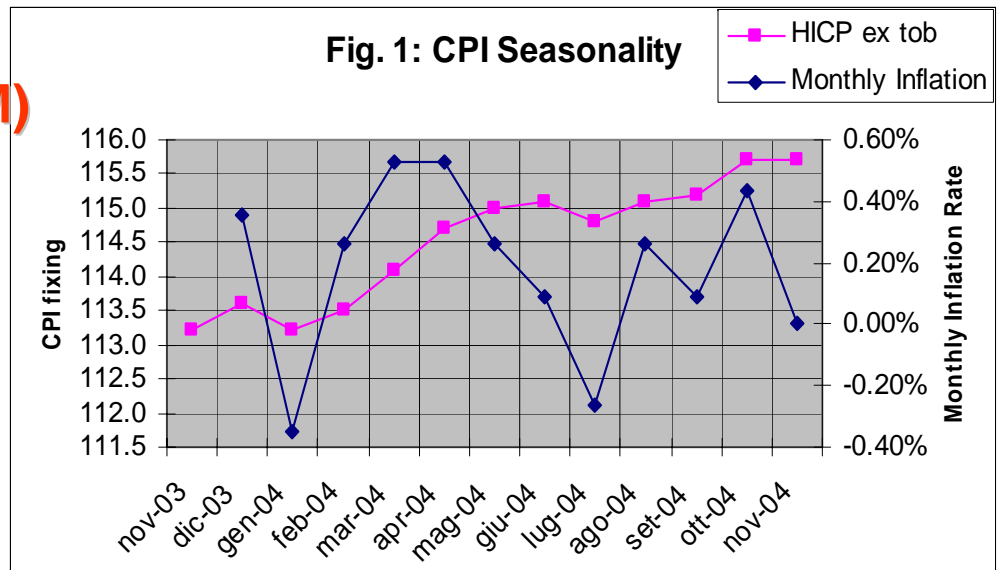
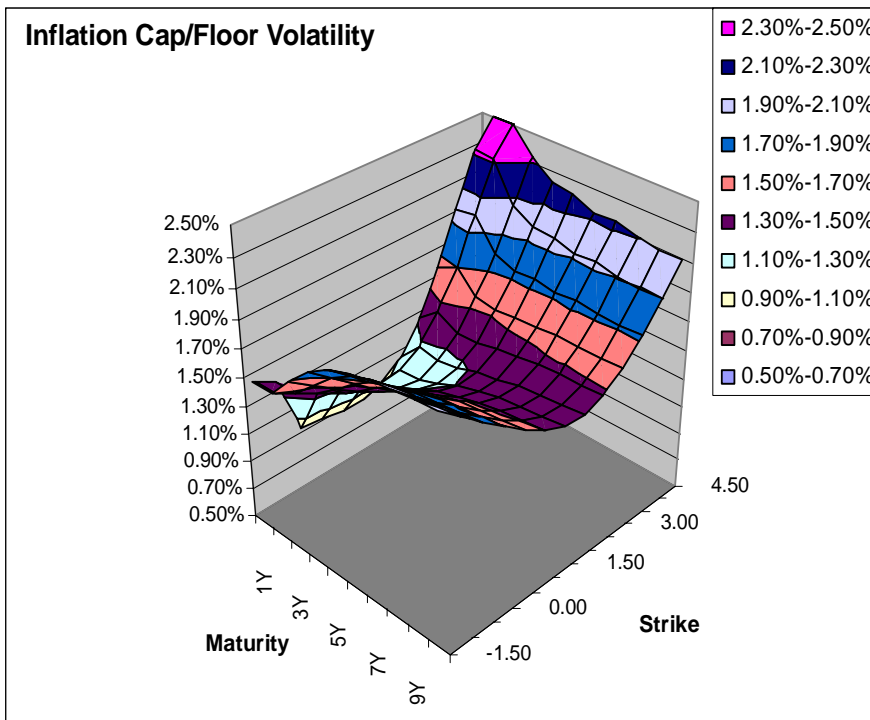
Market Rates				Zero Rates			
Instrument	Expiry	Bid	Ask	Date	Bid	Ask	Mid
Depo							
MB EUR0ND=	10/02/05	1.92500	1.97500	10/02/05	2.10382%	2.15451%	2.12917%
MB EUR5ND=	11/02/05	2.07500	2.12500	11/02/05	2.10376%	2.15445%	2.12910%
MB EUR6ND=	12/02/05	2.07500	2.12500	12/02/05	2.10340%	2.15407%	2.12873%
MB EUR9ND=	03/03/05	2.07500	2.12500	10/03/05	2.10212%	2.15274%	2.12743%
MB EUR1MD=	10/03/05	2.07500	2.12500	16/03/05	2.10744%	2.15804%	2.13274%
MB EUR2MD=	11/04/05	2.10500	2.15500	15/06/05	2.13935%	2.19062%	2.16499%
MB EUR3MD=	10/05/05	2.11500	2.16500	21/09/05	2.18485%	2.23499%	2.20947%
MB EUR6MD=	10/08/05	2.15500	2.20500	21/12/05	2.23762%	2.28794%	2.26273%
				15/03/06	2.29003%	2.34044%	2.32124%
				21/06/06	2.36189%	2.41229%	2.38709%
				20/09/06	2.42375%	2.47414%	2.44894%
				20/12/06	2.48378%	2.53425%	2.50906%
MB EURFUT1=	16/03/05	2.12750	2.17750	21/03/07	2.54405%	2.59443%	2.56924%
MB EURFUT2=	15/06/05	2.21750	2.26750	11/02/08	2.71457%	2.76332%	2.73895%
MB EURFUT3=	21/09/05	2.34250	2.39250	10/02/09	2.87057%	2.91930%	2.89493%
MB EURFUT4=	21/12/05	2.48750	2.53750	10/02/10	3.01027%	3.05904%	3.03466%
MB EURFUT5=	15/03/06	2.60250	2.65250	10/02/11	3.13615%	3.18498%	3.16057%
MB EURFUT6=	21/06/06	2.73250	2.78250	10/02/12	3.25113%	3.30003%	3.27559%
MB EURFUT7=	20/09/06	2.84250	2.89250	11/02/13	3.35485%	3.40395%	3.37940%
MB EURFUT8=	20/12/06	2.96250	3.01250	10/02/14	3.44720%	3.49625%	3.47173%
MB EURFUT9=	21/03/07			10/02/15	3.52763%	3.57667%	3.55210%
				10/02/16	3.59539%	3.64461%	3.62000%
				10/02/17	3.65188%	3.70116%	3.67653%
				12/02/18	3.69519%	3.74452%	3.71986%
				11/02/19	3.73397%	3.78378%	3.76408%
				10/02/20	3.76476%	3.83426%	3.80615%
				10/02/21	3.81064%	3.86017%	3.83541%
				10/02/22	3.83805%	3.88763%	3.86285%
				10/02/23	3.86637%	3.91602%	3.89120%
				12/02/24	3.89575%	3.94547%	3.92061%
				10/02/25	3.92625%	3.97507%	3.95019%
				10/02/26	3.95724%	3.99707%	3.96216%
				10/02/27	3.98944%	3.99906%	3.97477%
				10/02/28	3.99305%	4.01295%	3.98300%
				12/02/29	3.97654%	4.02647%	4.00151%
				11/02/30	3.99092%	4.04090%	4.01591%

One model

D) Calibration [2]



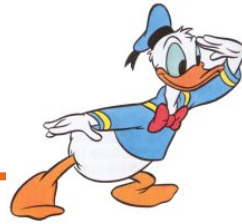
- Forecast CPI (Econ. Dept.):
 - short term curve (1M-12M)
- Seasonality effects



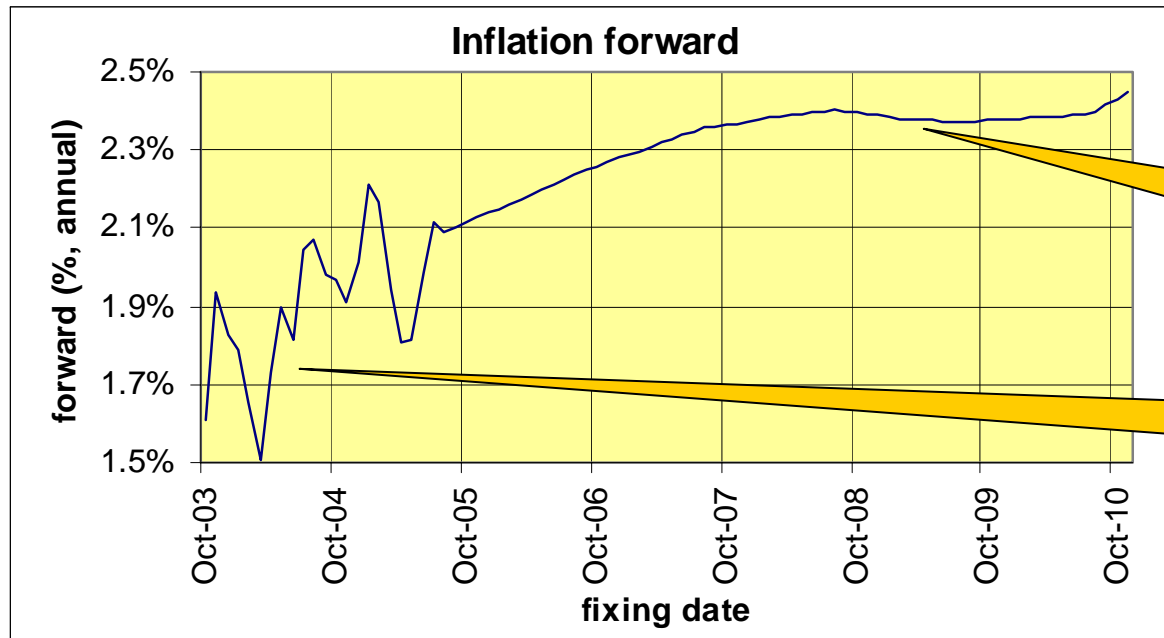
- Mkt inflation cap/floor calibration: “two regimes” market -> **two factor model**

Results

A) Forward curve



& pricing



No seasonality!

Yes seasonality!

Expected values at settlement date

$$E_{\text{settl}}^{30/01/05} \left[\frac{X_{\text{Oct.04}}}{X_{\text{Oct.03}}} - 1 \right] + E_{\text{settl}}^{30/01/05} [\text{option}]$$

$$E_{\text{settl}}^{30/01/06} \left[\frac{X_{\text{Oct.05}}}{X_{\text{Oct.04}}} - 1 \right] + E_{\text{settl}}^{30/01/06} [\text{option}]$$

Once the model is defined and calibrated, we are able to calculate expected values

Results

B) Greek letters



& bookkeeping

■ Inflation delta (double):
$$\frac{dNPV}{dR_j^{ZC}} = \sum_{\alpha=1}^N \left[\frac{\partial NPV}{\partial \sigma_\alpha} \frac{\partial \sigma_\alpha}{\partial R_j^{ZC}} + \frac{\partial NPV}{\partial \mu_\alpha} \frac{\partial \mu_\alpha}{\partial R_j^{ZC}} \right] \rightarrow \text{YoY}$$

■ Nominal delta:

$$\frac{dNPV}{dr_i} = \frac{\partial NPV}{\partial r_i} + \left[\frac{\partial NPV}{\partial k} \frac{\partial k}{\partial r_i} + \frac{\partial NPV}{\partial \xi} \frac{\partial \xi}{\partial r_i} \right] + \sum_{\alpha=1}^{N_{IPS}} \left[\frac{\partial NPV}{\partial \sigma_\alpha} \times \left(\frac{\partial \sigma_\alpha}{\partial r_i} + \frac{\partial \sigma_\alpha}{\partial k} \frac{\partial k}{\partial r_i} + \frac{\partial \sigma_\alpha}{\partial \xi} \frac{\partial \xi}{\partial r_i} \right) + \frac{\partial NPV}{\partial \mu_\alpha} \times \left(\frac{\partial \mu_\alpha}{\partial r_i} + \frac{\partial \mu_\alpha}{\partial k} \frac{\partial k}{\partial r_i} + \frac{\partial \mu_\alpha}{\partial \xi} \frac{\partial \xi}{\partial r_i} \right) \right]$$

■ Nominal cap/floor vega: ...

■ Theta (time value): ...

Expected Vs realized P&L

Node	Maturity	ZC swap curve	YoY swap curve
1M	12/01/04	0	0
2M	11/02/04	0	0
3M	11/03/04	97	0
4M	13/04/04	1,486	-1,407
5M	11/05/04	3,456	-3,168
6M	11/06/04	10,056	-5,337
7M	12/07/04	8,541	-7,635
8M	11/08/04	16,278	-9,996
9M	13/09/04	12,305	-12,613
10M	11/10/04	15,390	-14,870
11M	11/11/04	16,833	-17,401
12M	13/12/04	--	-19,985
1Y	13/12/04	19,321	--
2Y	12/12/05	-12,793	-43
3Y	11/12/06	1,706	-376
4Y	11/12/07	-4,945	-764
5Y	11/12/08	776	-1,161
6Y	11/12/09	1,197	-1,553
7Y	13/12/10	-7,407	-1,934
8Y	12/12/11	9,148	-2,299

Results

C) Expected Vs realised portfolio P&L



- **Realised profit & loss:** $\Delta NPV^{real} = NPV_f^{real} - NPV_i \quad t_f > t_i$
- **Expected profit & loss:** $\Delta NPV^{exp} = NPV_f^{exp} - NPV_i \quad t_f > t_i$

Exp. inflation P&L:
$$\Delta NPV = \sum_{j=1}^{N_R} \frac{\partial NPV}{\partial R_j} \Delta R_j = \sum_{j=1}^{N_R} \left[\frac{\partial NPV}{\partial R_j^{ZC}} \Delta R_j^{ZC} + \frac{\partial NPV}{\partial R_j^{YoY}} \Delta R_j^{YoY} \right]$$

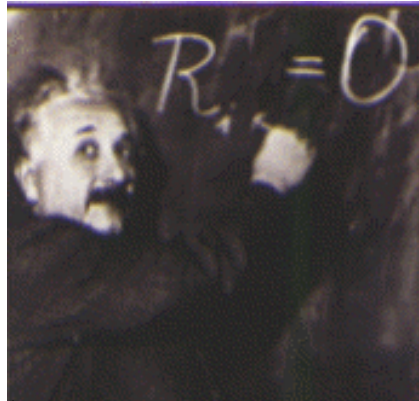
+ Exp. nominal P&L:
$$\Delta NPV = \sum_{k=1}^{N_r} \frac{\partial NPV}{\partial r_k} \Delta r_k, \quad \Delta r_i = r_k^f - r_k^i$$

+ Exp. Vega P&L + exp. theta P&L + ...

Total expected P&L

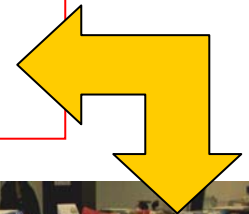
From theory to trading

A) The process &.....the characters



- Map business into equations
- Scan literature
- Build a model
- Software implementation
- Prototypes release & test
- Documentation

*Keyword:
time to market*



- Software engineering on target platform
- Documentation
- User training & help desk
- **Market evolution...**

*Keyword:
industrial standards*



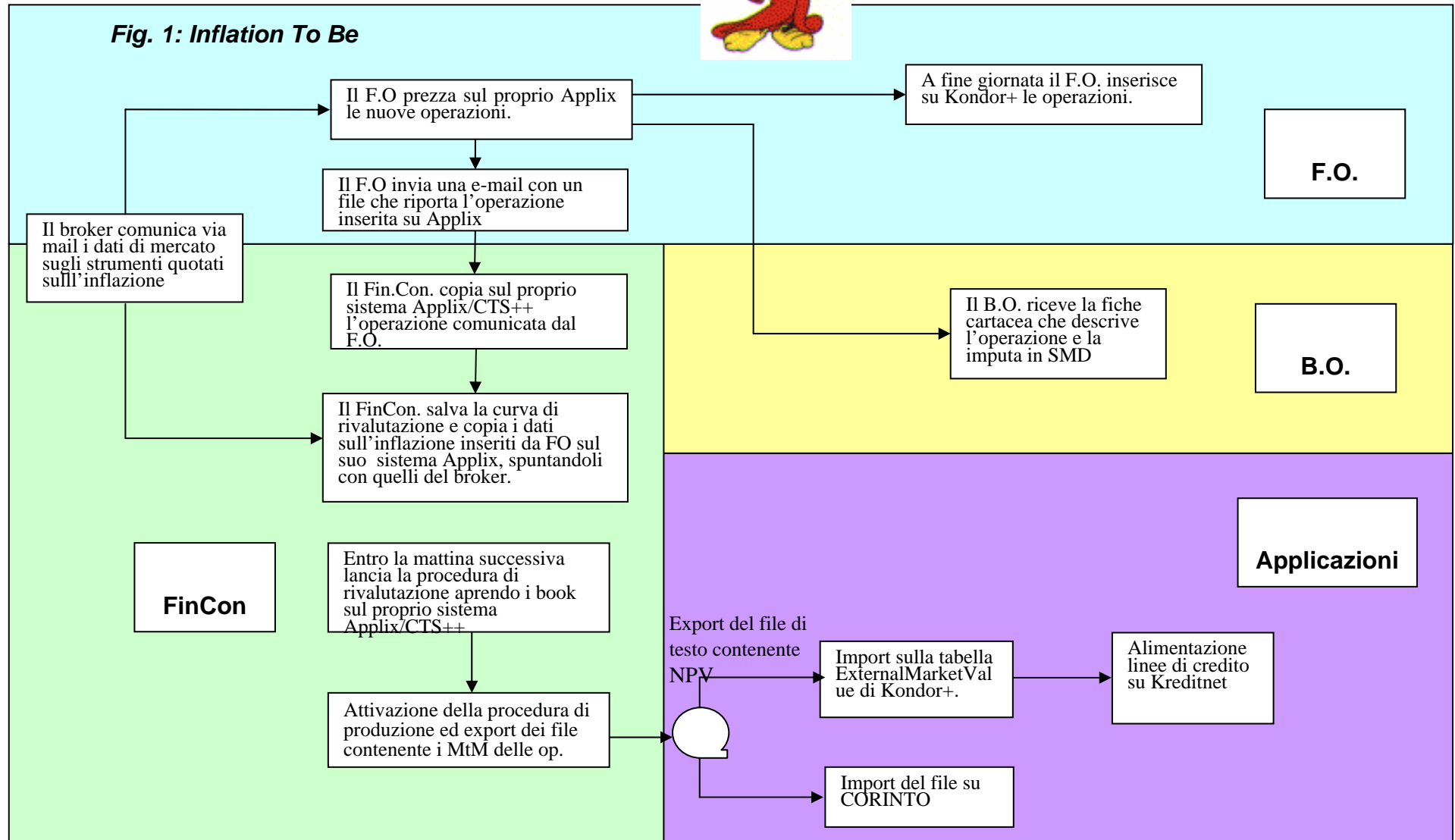
Financial Engineering, Front Office, Economic Dept., Risk Management, Financial Control, Back Office, IT, Organisation, Management (CNP)

From theory to trading

B) Organisation



Fig. 1: Inflation To Be



From theory to trading

C) Computing



- Problem: *performances*
- Solutions:
 - *improve the model*
 - *optimise the sw*
 - *empower the hw*
 - *Improve computing technology*

Grid computing = distributed computing + parallel computing



From theory to trading

D) Platforms, technologies & ...



■ Spreadsheet/C++ development:

- flexible as prototype
- poor performance and stability scaling
- high operational risk
- not supported

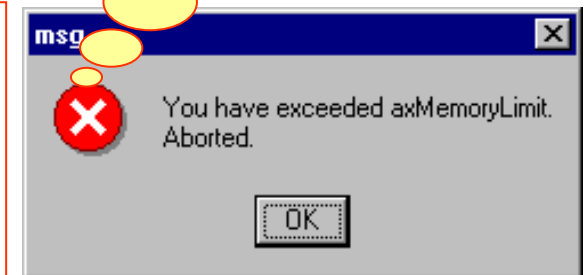
■ Integration with standard platforms (STP):

- STP (weak, strong...)
- supported (consultants)
- rigid & expensive

■ In house development of proprietary platforms:

- in house development
- flessibile for product innovation
- expensive

**Technological issues
must not be
underestimated...**

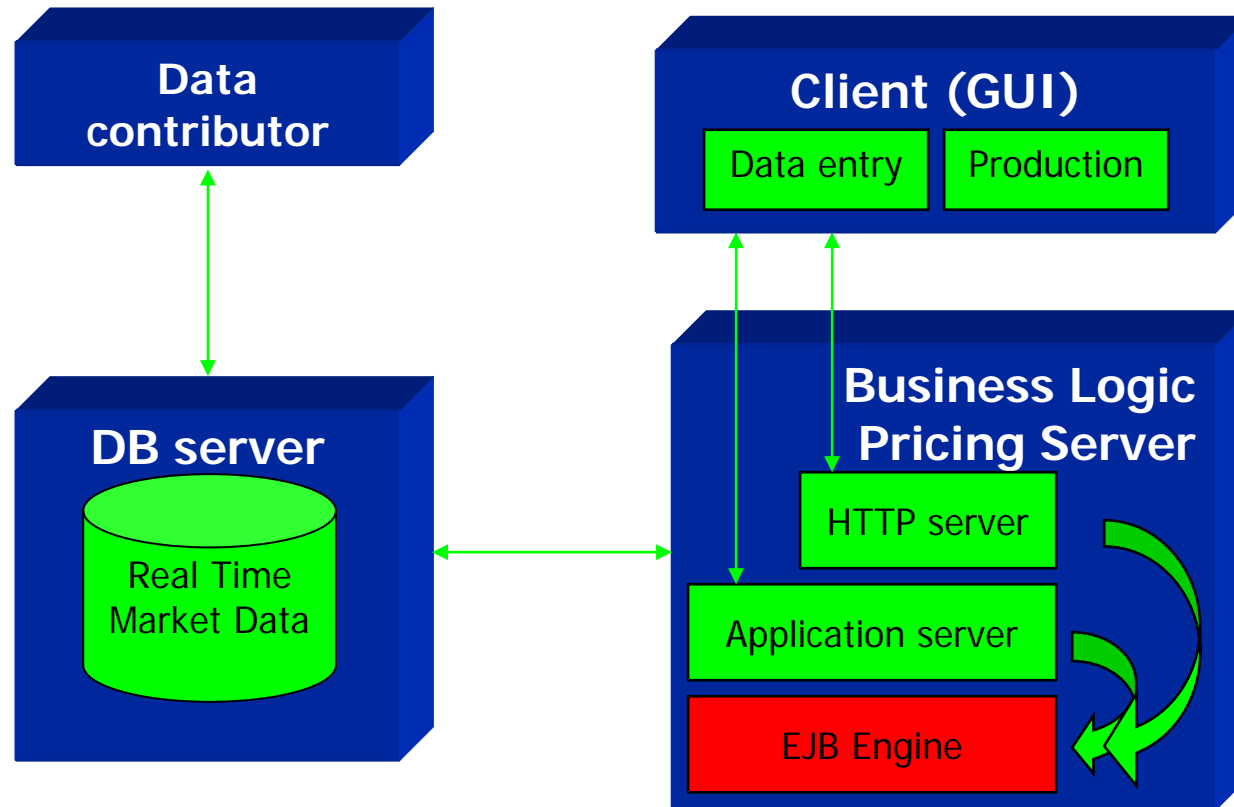


From theory to trading

E) ... architectures



Client - server architectures:



References



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- [2] F. Mercurio, “*Pricing of inflation-Linked derivatives*”, Quant Congress Europe 2004, London, 8-9 November 2004 (<http://www.fabiomercurio.it>)
- [3] Caboto Internal Reports:
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Banca Caboto Financial Engineering Unit:

- Derivatives Modeling: *M. Bianchetti, C. Duminuco, S. Frasson, M. Pucci*
- Derivatives Technology: *V. Miccoli, A. Marcarini, L. Perini, S. Ramolfo, S. Mancarella, E. Piccolini*