Caboto

PRICING & HEDGING INTEREST RATE DERIVATIVES

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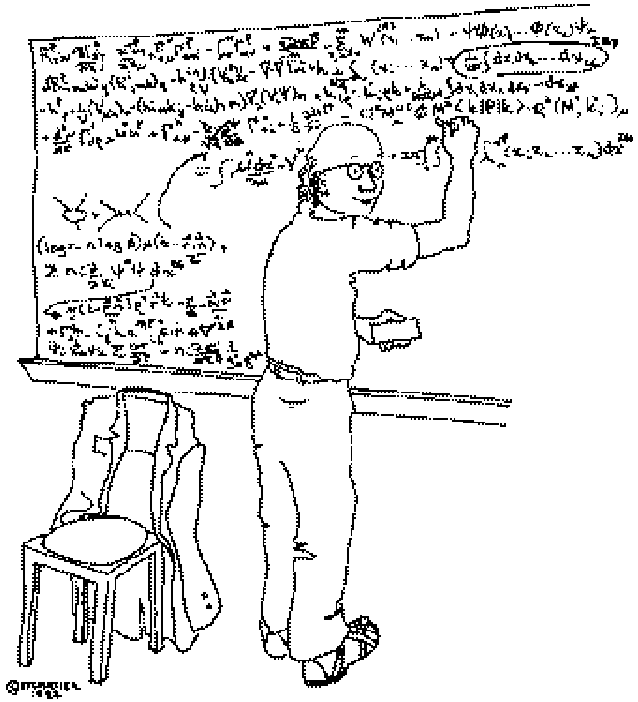
Problemi di matematica dagli istituti finanziari - Bologna, 22 febbraio 2006

Summary

- **Definition of the business problem**
  - Example: "*Schermo Totale*"
- **One model**
  - Basic assumptions
  - Solution in closed form
  - Calibration
- **Results**
  - Forward curve & pricing
  - Greek letters & bookeeping
- **From theory to trading**
  - The process & the characters
  - Organisation
  - Computing
  - Platforms, technologies, architectures
- **References**

Some math will be necessary...



"At this point we notice that this equation is beautifully simplified if we assume that space-time has 92 dimensions."

Definition of the business problem

A) Example: "*Schermo Totale*" - termsheet

- **Deal characteristics:**
  - **start:** 30 Jan. 2003
  - **maturity:** 30 Jan. 2006 (3Y)
  - **nominal:** 2×10^8 €
  - **underlying CPI:** *HICP unrevised (Bloomberg: CPALEMU)*
  - **reference CPI fixing** : Oct. 2003
- **Counterparty A pays to counterparty B:**
  - upfront: XXX % *(unknown to be determined)*
  - I coupon: 3.5% *(% annual, act/act, unadj.)*
  - II+III coupon: 0.80% + MAX[0%; 100% x ($HICP_f$ / $HICP_i$-1)] *(idem)*
- **Counterparty B pays to counterparty A:**
  - Euribor 3M + 24 bps *(% quarterly, act/360, adj.)*

Here is the income…

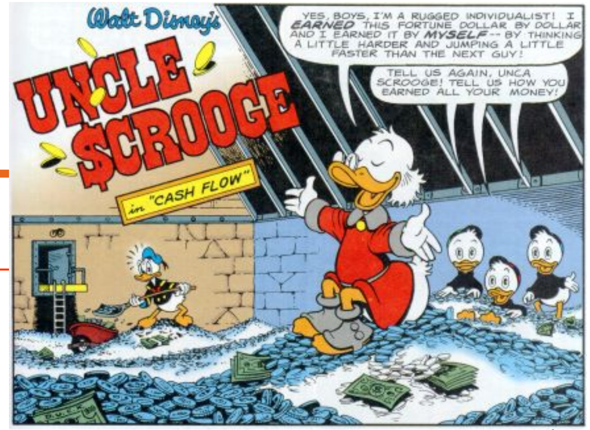
…and here is the risk!

***"Structuration"*:**
building up tailor made financial instruments with "simple" bricks

Payoff = Plain vanilla swap + inflation-linked swap + opzione

Definition of the business problem

B) "*Schermo Totale*" - inflation-linked cashflows



Inflation-linked cashflows

- maturity cashflows:
  - 30 Jan. 2005: $\mathrm{Max}\left[X_{oct.04} / X_{oct.03} - 1 \, ; \, 0 \right]$
  - 30 Jan. 2006: $\mathrm{Max}\left[X_{oct.05} / X_{oct.04} - 1 \, ; \, 0 \right]$

Now we need a model!

- Expected cashflows at *settlement date* (*NPV*):

  Expected value — Expected value

  - 30 Jan. 2005:

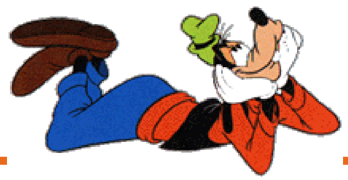
$$d_{settl}(30/01/05) \times \left\{ E_{settl}^{30/01/05}\left[X_{Oct.04} / X_{Oct.03} - 1 \right] + E_{settl}^{30/01/05}[\text{option}] \right\}$$

  - 30 Jan. 2006:

$$d_{settl}(30/01/06) \times \left\{ E_{settl}^{30/01/06}\left[X_{Oct.05} / X_{Oct.04} - 1 \right] + E_{settl}^{30/01/06}[\text{option}] \right\}$$

One model

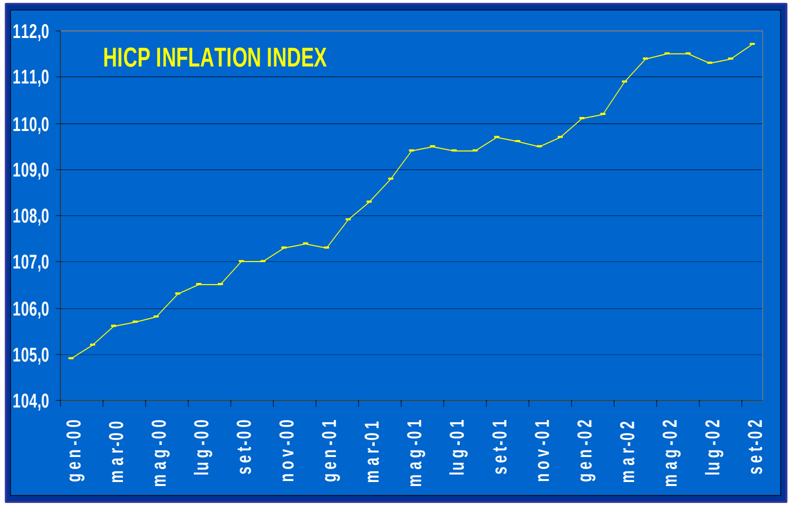
A) Basic assumptions



- **Assumptions on the underlying CPI X_t:**
  - ***non-tradable***
  - ***monthly* fixing with *lag* δ**
  - ***stochastic continuous* process**
  - ***BS-type* diffusion dynamic:**

$$\frac{dY_t}{Y_t} = \mu(t)\,dt + \sigma(t)\,dW_t, \quad Y_t = X_{t-\delta}$$

drift — volatility — brownian motion



- **Assumptions on the interest rate r_t:**
  - **$\exists$ a *bank account B(t)*, a *spot Martingale measure* and a *short rate* r_t s.t. $\forall$ T>t , $\forall$ δ_c>δ:**

Exp. value at t — short rate

$$\frac{d_t(T)}{B(t)} = E_t\left[\frac{1}{B(T)}\right] = E_t\left[\exp\left(-\int_t^T r_u\,du\right)\right]$$

$$P_t(T,\delta_c) = B(t)\,E_t\left[\exp\left(-\int_t^T r_u\,du\right) X_{T-\delta_c}\right]$$

forward contract — unknown

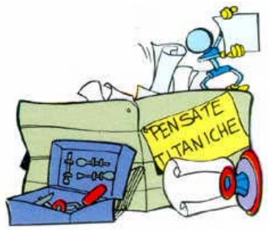
  - ***Hull-White* diffusion dynamic:**

$$dr_t = \kappa\left[\vartheta(t) - r_t\right]dt + \xi\,dW_t$$

mean reversion — volatility — brownian motion

One model

B) Solution in closed form



N stochastic factors

- **from spot to T-forward measure:**

$$\begin{cases} \dfrac{dY_t}{Y_t} = \mu^T(t)\,dt + \sigma(t)\cdot dW_t^T, & dW_t^T = dW_t + \xi\, B(t,T)\,dt \\ \mu^T(t) = \mu(t) - \xi\sigma(t)B(t,T), \end{cases}$$

- **Zero coupon price:**

End fixing, unknown — *Start fixing, known* — *Last published fixing, known*

$$P_t(T,\delta_c) = d_t(T)\left(\frac{1}{X_{t-\delta_c}} E_t^T\left[X_{T-\delta_c}\right] - 1\right) = D_t(T)\left[\frac{X_{t-\delta}}{X_{t-\delta_c}} e^{\int_t^{\hat{T}} \mu^T(u)du} - 1\right]$$

- **Forward YoY coupon price:**

Start & end fixing, unknown — *Convexity adjustment, dipends on volatility only*

$$P_t(S,T,\delta_c) = d_t(T)E_t^T\left[\frac{X_{T-\delta_c}}{X_{S-\delta_c}} - 1\right] = d_t(T)\left[\frac{P_t(T,\delta_c)D_t(S)}{P_t(S,\delta_c)D_t(T)} e^{-C_t(S,T,\delta_c,\sigma)} - 1\right]$$

- **Call/put option price:**

$$P_t(T,\delta_c,K,\chi) = d_t(T)\text{Max}\left[\chi\left(X_{T-\delta_c},K\right);0\right] = \ldots \quad (\chi = \pm 1)$$

- **Floored YoY coupon price:**

$$P_t(S,T,\delta_c,K) = d_t(T)\text{Max}\left[\frac{X_{T-\delta_c}}{X_{S-\delta_c}} - 1, K\right] = \ldots$$

One model

C) Calibration [1]

Bootstrapping!

- **1st step: short rate calibration:**
  - **nominal mkt rates** $[r_{mkt} \rightarrow \theta]$
  - **nominal mkt cap/floor vols** $[\sigma_{cap/floor} \rightarrow \kappa, \xi]$
- **2nd step: CPI volatility calibration:**
  - **mkt YoY inflation rates** $[R_{YoY} \rightarrow \sigma(t)]$
- **3rd step: CPI drift calibration:**
  - **mkt ZC inflation rates** $[R_{ZC} \rightarrow \mu(t)]$

HICP EU all items									
Mkt ZC Swap rates				Mkt YoY Swap Rates				Basis CPI	
Node	Bid (%)	Ask (%)	Mid (%)	Node	Bid (%)	Ask (%)	Mid (%)	Date	01/11/04
1	5.2782	5.2782	5.2782	1	5.2782	5.2782	5.2782	Fixing	116.4
2	-0.5059	-0.5059	-0.5059	2	-0.4888	-0.4888	-0.4888	Mkt calendar	EURO
3	1.0349	1.0349	1.0349	3	1.0417	1.0417	1.0417	Mkt daycount	2
4	2.3376	2.3376	2.3376	4	2.3202	2.3202	2.3202	Delay min.	59
5	3.0997	3.0997	3.0997	5	3.0606	3.0606	3.0606		
6	2.7680	2.7680	2.7680	6	2.7387	2.7387	2.7387		
7	2.4927	2.4927	2.4927	7	2.4708	2.4708	2.4708		
8	1.9392	1.9392	1.9392	8	1.9321	1.9321	1.9321		
9	1.7219	1.7219	1.7219	9	1.7190	1.7190	1.7190		
10	1.7434	1.7434	1.7434	10	1.7389	1.7389	1.7389		
11	1.8759	1.8759	1.8759	11	1.8673	1.8673	1.8673		
12	1.8000	1.8000	1.8000	12	1.7929	1.7929	1.7929		
1	1.8000	1.8000	1.8000	1	1.8000	1.8000	1.8000		
2	2.1300	2.1300	2.1300	2	2.1300	2.1300	2.1300		
3	2.1600	2.1600	2.1600	3	2.1575	2.1575	2.1575		
4	2.1900	2.1900	2.1900	4	2.1875	2.1875	2.1875		
5	2.2300	2.2300	2.2300	5	2.2250	2.2250	2.2250		
6	2.2400	2.2400	2.2400	6	2.2350	2.2350	2.2350		
7	2.2700	2.2700	2.2700	7	2.2625	2.2625	2.2625		
8	2.2900	2.2900	2.2900	8	2.2825	2.2825	2.2825		
9	2.3100	2.3100	2.3100	9	2.3025	2.3025	2.3025		
10	2.3400	2.3400	2.3400	10	2.3300	2.3300	2.3300		
15	2.4200	2.4200	2.4200	15	2.4000	2.4000	2.4000		
20	2.5100	2.5100	2.5100	20	2.4650	2.4650	2.4650		
25	2.5700	2.5700	2.5700	25	2.5050	2.5050	2.5050		
30	2.6100	2.6100	2.6100	30	2.5375	2.5375	2.5375		

Market Rates			
Instrument	Expiry	Bid	Ask
Depo			
MB EUROND=	10/02/05	1.92500	1.97500
MB EURSND=	11/02/05	2.07500	2.12500
MB EURSWD=	17/02/05	2.07500	2.12500
MB EUR1MD=	10/03/05	2.07500	2.12500
MB EUR2MD=	11/04/05	2.10500	2.15500
MB EUR3MD=	10/05/05	2.11500	2.16500
MB EUR6MD=	10/08/05	2.15500	2.20500
Fix			
--	--	0.00000	0.00000
Future			
MB EURFUT1=	16/03/05	2.12750	2.17750
MB EURFUT2=	15/06/05	2.21750	2.26750
MB EURFUT3=	21/09/05	2.34250	2.39250
MB EURFUT4=	21/12/05	2.48750	2.53750
MB EURFUT5=	15/03/06	2.60250	2.65250
MB EURFUT6=	21/06/06	2.73250	2.78250
MB EURFUT7=	20/09/06	2.84250	2.89250
MB EURFUT8=	20/12/06	2.96250	3.01250
MB EURFUT9=	21/03/07		
Swap			
MB EUR3YS=	11/02/08	2.74400	2.79400
MB EUR4YS=	10/02/09	2.90100	2.95100
MB EUR5YS=	10/02/10	3.03800	3.08800
MB EUR6YS=	10/02/11	3.16000	3.21000
MB EUR7YS=	10/02/12	3.27000	3.32000
MB EUR8YS=	11/02/13	3.36900	3.41900
MB EUR9YS=	10/02/14	3.45500	3.50500
MB EUR10YS=	10/02/15	3.52900	3.57900
MB EUR11YS=	10/02/16	3.59100	3.64100
MB EUR12YS=	10/02/17	3.64300	3.69300
MB EUR15YS=	10/02/20	3.76100	3.81100
MB EUR20YS=	10/02/25	3.88200	3.93200
MB EUR25YS=	11/02/30	3.94000	3.99000
MB EUR30YS=	12/02/35	3.96000	4.01000
MB EUR40YS=	10/02/45	3.98100	4.03100

Zero Rates			
Date	Bid	Ask	Mid
10/02/05	2.10382%	2.15451%	2.12917%
11/02/05	2.10376%	2.15445%	2.12910%
17/02/05	2.10340%	2.15407%	2.12873%
10/03/05	2.10212%	2.15274%	2.12743%
16/03/05	2.10744%	2.15804%	2.13274%
15/06/05	2.13935%	2.18982%	2.16458%
21/09/05	2.18426%	2.23469%	2.20947%
21/12/05	2.23752%	2.28794%	2.26273%
15/03/06	2.29603%	2.34645%	2.32124%
21/06/06	2.36189%	2.41229%	2.38709%
20/09/06	2.42375%	2.47414%	2.44894%
20/12/06	2.48387%	2.53425%	2.50906%
21/03/07	2.54405%	2.59443%	2.56924%
11/02/08	2.71457%	2.76332%	2.73895%
10/02/09	2.87057%	2.91930%	2.89493%
10/02/10	3.01027%	3.05904%	3.03466%
10/02/11	3.13615%	3.18498%	3.16057%
10/02/12	3.25113%	3.30003%	3.27559%
11/02/13	3.35486%	3.40382%	3.37934%
10/02/14	3.44720%	3.49625%	3.47173%
10/02/15	3.52753%	3.57667%	3.55210%
10/02/16	3.59539%	3.64461%	3.62000%
10/02/17	3.65188%	3.70116%	3.67653%
12/02/18	3.69519%	3.74452%	3.71986%
11/02/19	3.73937%	3.78878%	3.76408%
10/02/20	3.78476%	3.83426%	3.80951%
10/02/21	3.81064%	3.86017%	3.83541%
10/02/22	3.83805%	3.88763%	3.86285%
10/02/23	3.86637%	3.91602%	3.89120%
12/02/24	3.89575%	3.94547%	3.92061%
10/02/25	3.92528%	3.97509%	3.95019%
10/02/26	3.93724%	3.98707%	3.96216%
10/02/27	3.94984%	3.99969%	3.97477%
10/02/28	3.96306%	4.01295%	3.98800%
12/02/29	3.97654%	4.02647%	4.00151%
11/02/30	3.99092%	4.04090%	4.01591%

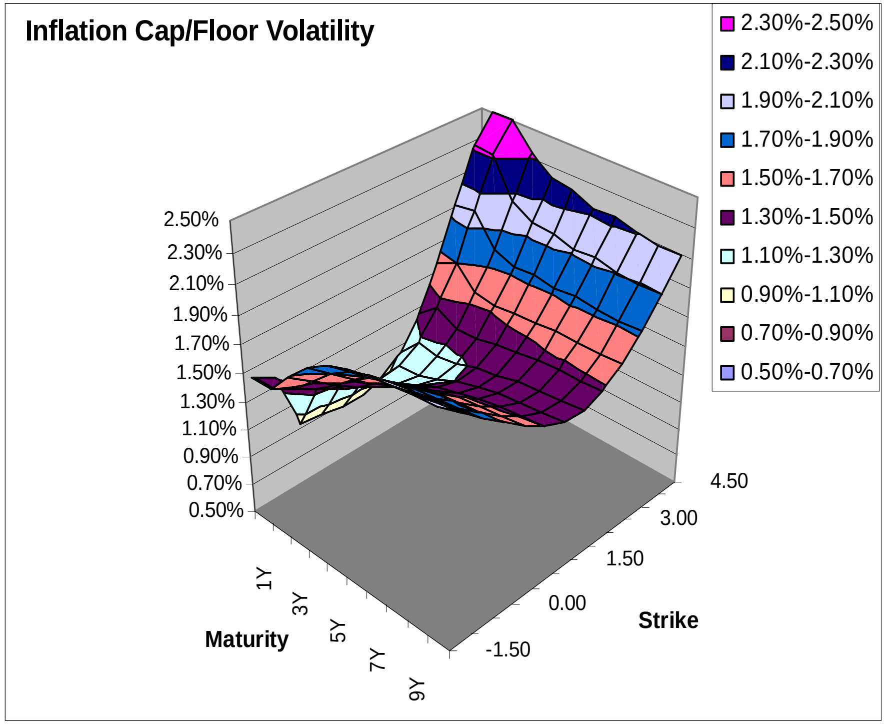
Output:

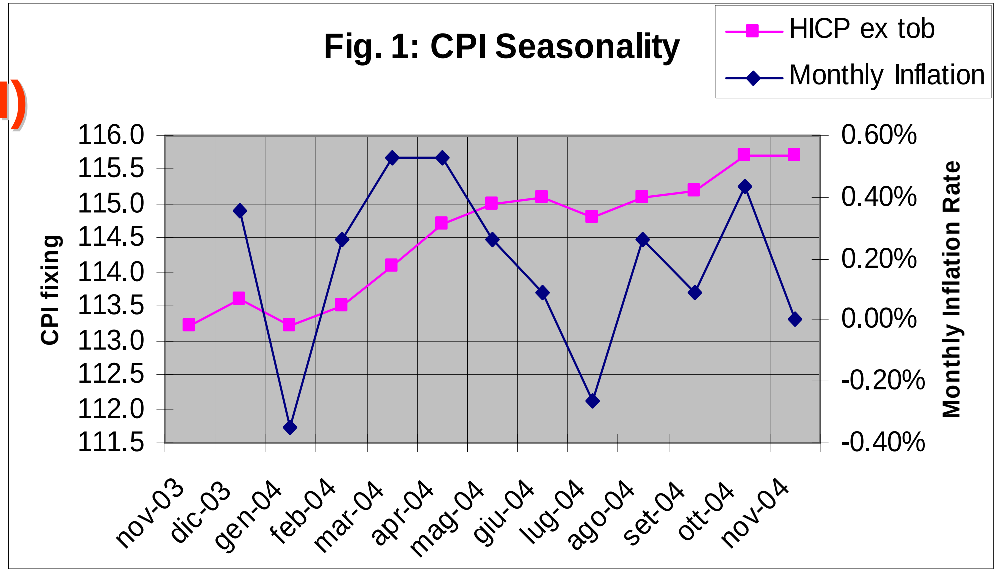
short rate mean reversion & volatility (H.W.) + CPI drift & volatility (B.S.)

One model
D) Calibration [2]

- Forecast CPI (Econ. Dept.):
  - short term curve (1M-12M)

- **Seasonality** effects





- **Mkt inflation cap/floor calibration: "two regimes" market -> two factor model**

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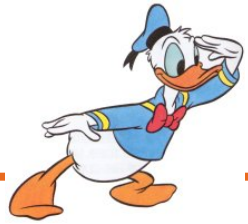
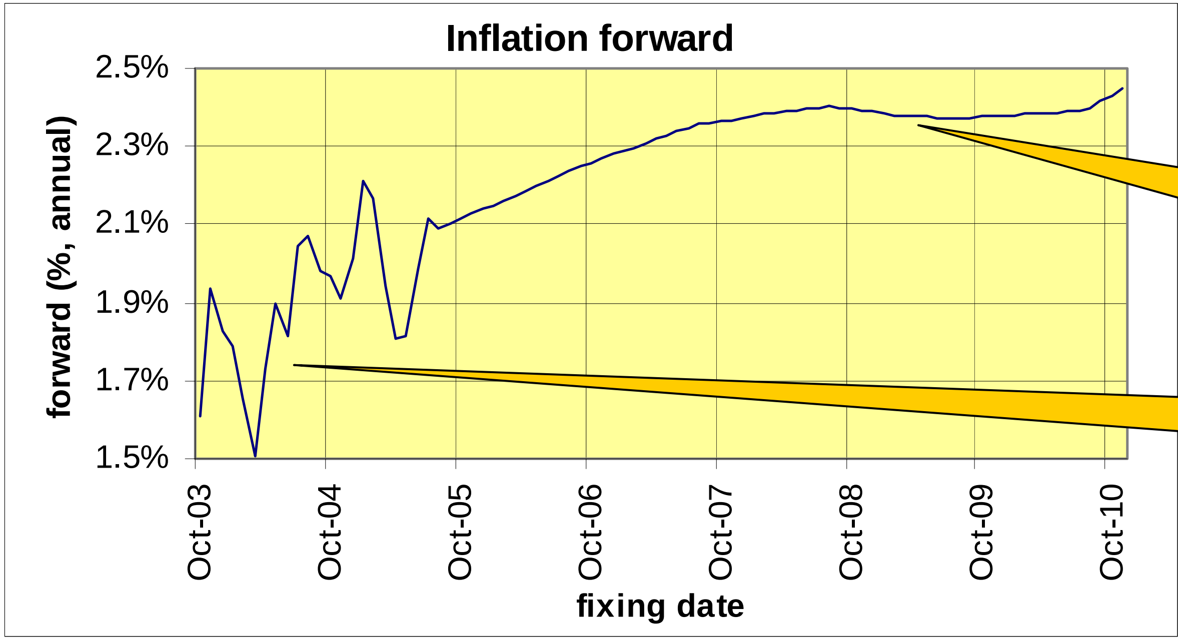
Results

A) Forward curve



& pricing



No seasonality!

Yes seasonality!

Expected values at settlement date

$$E^{30/01/05}_{settl}\left[X_{Oct.04}/X_{Oct.03} - 1\right] + E^{30/01/05}_{settl}\left[option\right]$$

$$E^{30/01/06}_{settl}\left[X_{Oct.05}/X_{Oct.04} - 1\right] + E^{30/01/06}_{settl}\left[option\right]$$

Once the model is defined and calibrated, we are able to calculate expected values

Results

B) Greek letters  & bookeeping

- **Inflation delta (double):** $$\frac{dNPV}{dR_j^{ZC}} = \sum_{a=1}^{N}\left[\frac{\partial NPV}{\partial \sigma_\alpha}\frac{\partial \sigma_\alpha}{\partial R_j^{ZC}} + \frac{\partial NPV}{\partial \mu_\alpha}\frac{\partial \mu_\alpha}{\partial R_j^{ZC}}\right] \rightarrow \text{YoY}$$

- **Nominal delta:**

$$\frac{dNPV}{dr_i} = \frac{\partial NPV}{\partial r_i} + \left[\frac{\partial NPV}{\partial k}\frac{\partial k}{\partial r_i} + \frac{\partial NPV}{\partial \xi}\frac{\partial \xi}{\partial r_i}\right] + \sum_{\alpha=1}^{N_{IPS}}\left[\begin{array}{l}\frac{\partial NPV}{\partial \sigma_\alpha}\times\left(\frac{\partial \sigma_\alpha}{\partial r_i} + \frac{\partial \sigma_\alpha}{\partial k}\frac{\partial k}{\partial r_i} + \frac{\partial \sigma_\alpha}{\partial \xi}\frac{\partial \xi}{\partial r_i}\right) + \\ \frac{\partial NPV}{\partial \mu_\alpha}\times\left(\frac{\partial \mu_\alpha}{\partial r_i} + \frac{\partial \mu_\alpha}{\partial k}\frac{\partial k}{\partial r_i} + \frac{\partial \mu_\alpha}{\partial \xi}\frac{\partial \xi}{\partial r_i}\right)\end{array}\right]$$

- **Nominal cap/floor vega: ...**

- **Theta (time value): ...**

Expected Vs realized P&L

Node	Maturity	ZC swap curve	YoY swap curve
1M	12/01/04	0	0
2M	11/02/04	0	0
3M	11/03/04	97	0
4M	13/04/04	1,486	-1,407
5M	11/05/04	3,456	-3,168
6M	11/06/04	10,056	-5,337
7M	12/07/04	8,541	-7,635
8M	11/08/04	16,278	-9,996
9M	13/09/04	12,305	-12,613
10M	11/10/04	15,390	-14,870
11M	11/11/04	16,833	-17,401
12M	13/12/04	--	-19,985
1Y	13/12/04	19,321	--
2Y	12/12/05	-12,793	-43
3Y	11/12/06	1,706	-376
4Y	11/12/07	-4,945	-764
5Y	11/12/08	776	-1,161
6Y	11/12/09	1,197	-1,553
7Y	13/12/10	-7,407	-1,934
8Y	12/12/11	9,148	-2,299

Results

C) Expected Vs realised portfolio P&L

- ***Realised* profit & loss:** $\Delta NPV^{real} = NPV_f^{real} - NPV_i \qquad t_f > t_i$
- ***Expected* profit & loss:** $\Delta NPV^{\exp} = NPV_f^{\exp} - NPV_i \qquad t_f > t_i$

Exp. inflation P&L:

$$\Delta NPV = \sum_{j=1}^{N_R} \frac{\partial \mathrm{NPV}}{\partial R_j} \Delta R_j = \sum_{j=1}^{N_R} \left[\frac{\partial \mathrm{NPV}}{\partial R_j^{\mathrm{ZC}}} \Delta R_j^{\mathrm{ZC}} + \frac{\partial \mathrm{NPV}}{\partial R_j^{\mathrm{YoY}}} \Delta R_j^{\mathrm{YoY}} \right]$$

+ Exp. nominal P&L:

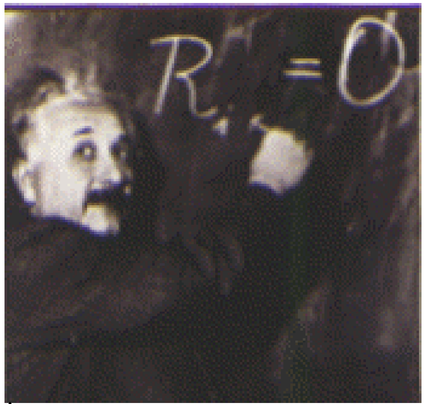
$$\Delta NPV = \sum_{k=1}^{N_r} \frac{\partial \mathrm{NPV}}{\partial r_k} \Delta r_k \ , \quad \Delta r_i = r_k^f - r_k^i$$

+ Exp. Vega P&L + exp. theta P&L + ...

Total expected P&L

From theory to trading

A) The process &…………………………..the characters



- Map business into equations
- Scan literature
- Build a model
- Software implementation
- Prototypes release & test
- Documentation

Keyword:
time to market

- Software engineering on target platform
- Documentation
- User training & help desk
- **Market evolution...**

Keyword:
industrial standards

Actors: Financial Engineering, Front Office, Economic Dept., Risk Man., Financial Control, Back Office, IT, Organisation, Management (CNP)

From theory to trading

B) Organisation

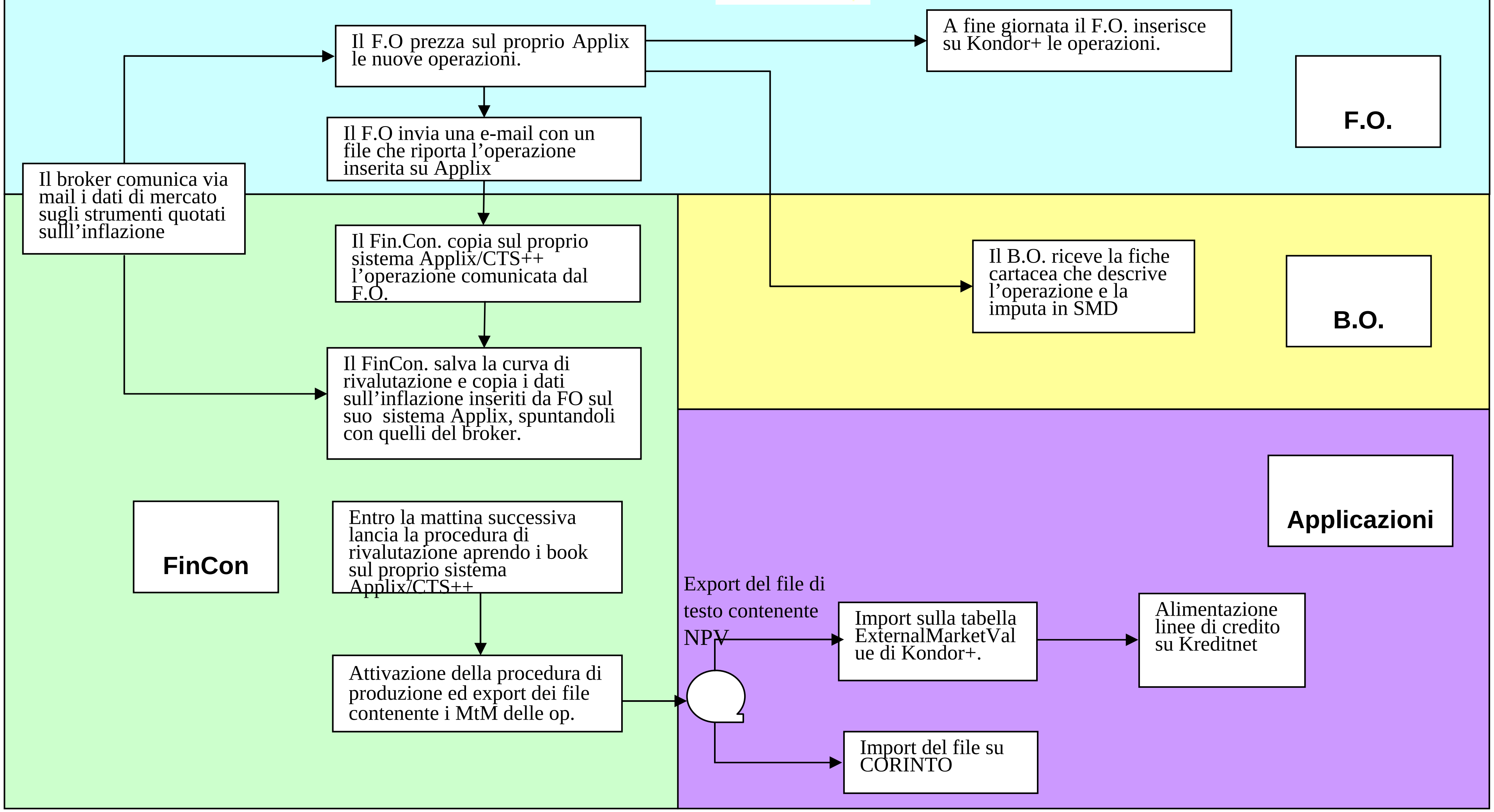


Fig. 1: Inflation To Be

From theory to trading

C) Computing

- Problem: ***performances***
- Solutions:
  - ***improve the model***
  - ***optimise the sw***
  - ***empower the hw***
  - ***Improve computing technology***

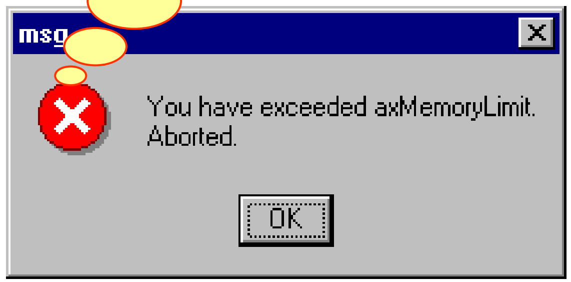
Grid computing = distributed computing + parallel computing

From theory to trading

D) Platforms, technologies & ...

- **Spreadsheet/C^{++} development**:
  - flexible as prototype
  - poor performance and stability scaling
  - high operational risk
  - not supported
- **Integration with standard platforms (STP)**:
  - STP (weak, strong…)
  - supported (consultants)
  - rigid & expensive
- **In house development of proprietary platforms**:
  - in house development
  - flessibile for product innovation
  - expensive

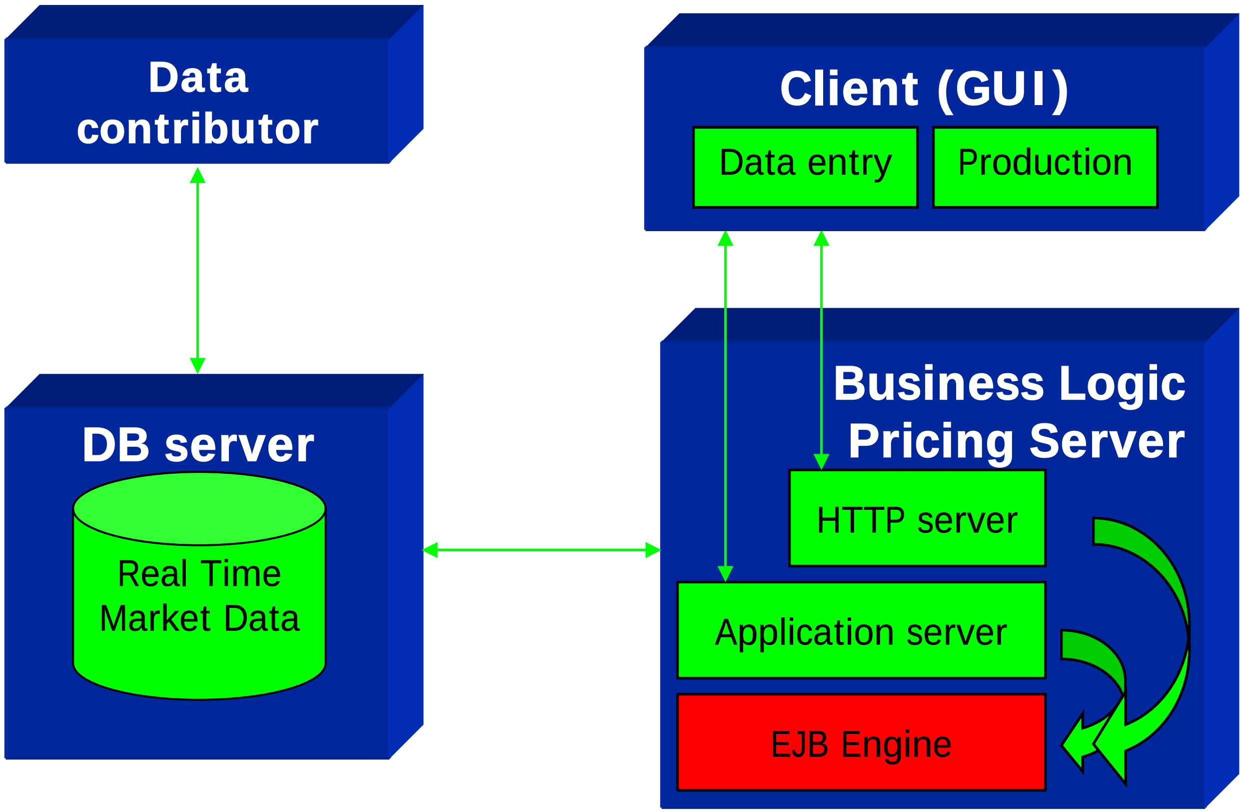
Technological issues must not be underestimated...



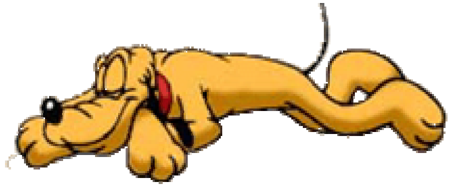
From theory to trading

E) … architectures

Client - server architectures:



References



[1] R. Jarrow, Y. Yldirim, "*Pricing TIPS and Related Derivatives Using an HJM Model*", Journal of Financial and Quantitative Analysis, Vol. 38, No. 2, June 2003 (http://sominfo.syr.edu/facstaff/yildiray)

[2] F. Mercurio, "*Pricing of inflation-Linked derivatives*", Quant Congress Europe 2004, London, 8-9 November 2004 (http://www.fabiomercurio.it)

[3] Caboto Internal Reports:

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- M. Bianchetti, "*Sistema Inflation-Linked Derivatives: implementazione*", marzo 2003
- D. Poggi, "*Gestione del Book Inflation-Linked Derivatives - procedura operativa*", aprile 2003.

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